

Effect of Prandtl number on flow stability

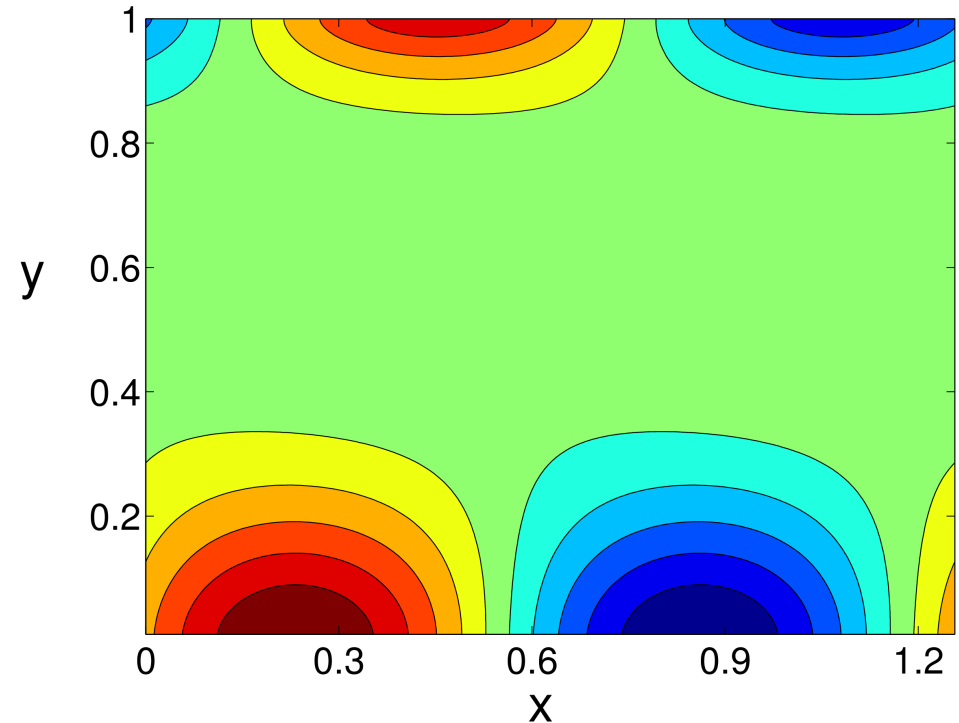


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Pressure perturbation contours during mode-crossing

Outline

- Motivation
- Model problem
- Governing equations for base flow and stability
- Results
- Conclusion and Future work

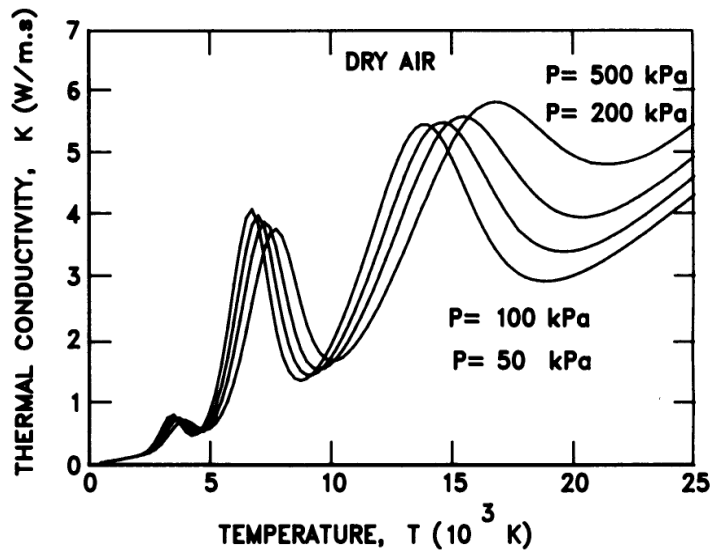
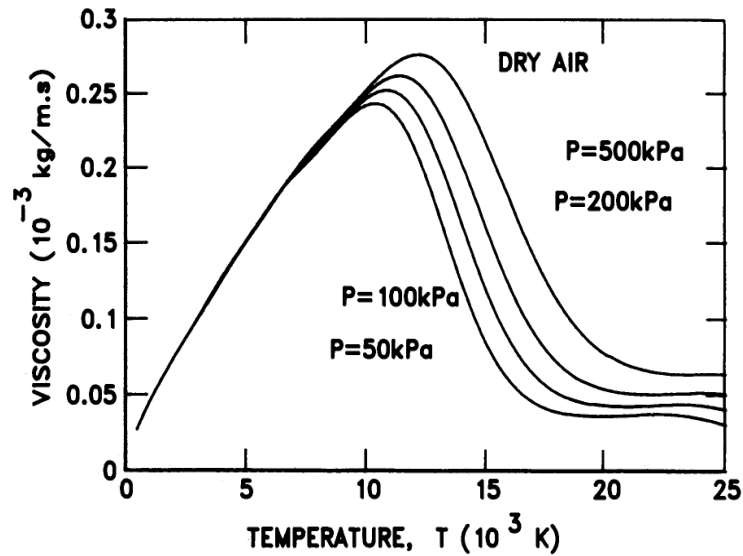
Previous works

For a compressible Couette flow :

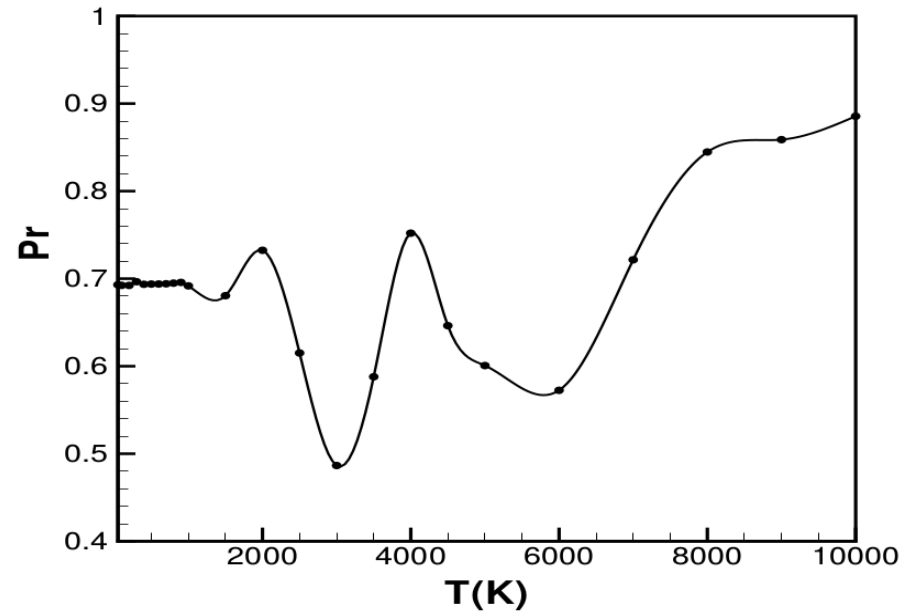
- Duck, Erlebacher and Hussaini (JFM 258, 1994)
 - Instabilities are found only in the inviscid limit
- Hu and Zhong (POF 10.3, 1998)
 - Destabilizing role of viscosity for a range of Re and wavenumber
- Malik, Dey and Alam (PRE 77.3, 2008)
 - Stratification of viscosity is stabilizing

→ A constant value of Prandtl number

Real scenario



For equilibrium air at 1 bar pressure



Independent variations in viscosity
and conductivity

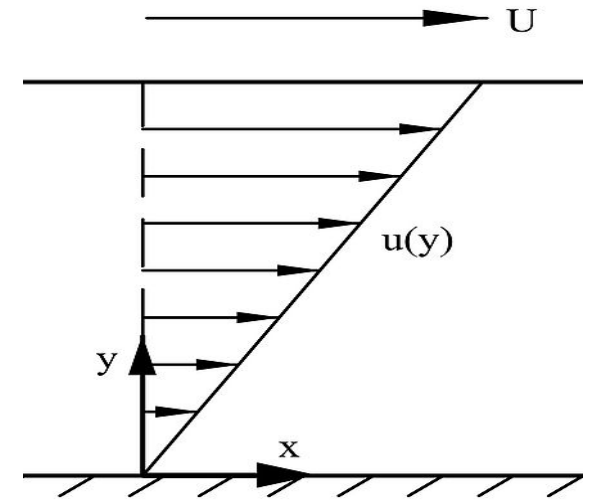
Large variation in Prandtl number

Model problem

For our study :

- Viscosity & conductivity constant
 - But their ratio i.e. **Pr** is varying
- Range of **Pr** taken from high temperature air

Vary between 0.2 to 0.9



Governing equations

3D NS equations



Non-dimensionalized by
the top wall quantities



Assumption of steady
& unidirectional flow



Constant viscosity &
conductivity (= 1)



x momentum : $\frac{du}{dy} = \text{constant}$

Energy : $\frac{d}{dy} \left(\frac{dT}{dy} \right) + (\gamma - 1) M^2 Pr \left(\frac{du}{dy} \right)^2 = 0$



No slip b.c.

Adiabatic bottom wall,
isothermal top wall



$$u = y$$

$$T = 1 + \frac{(\gamma - 1) Pr M^2}{2} (1 - y^2)$$

Base flow profiles

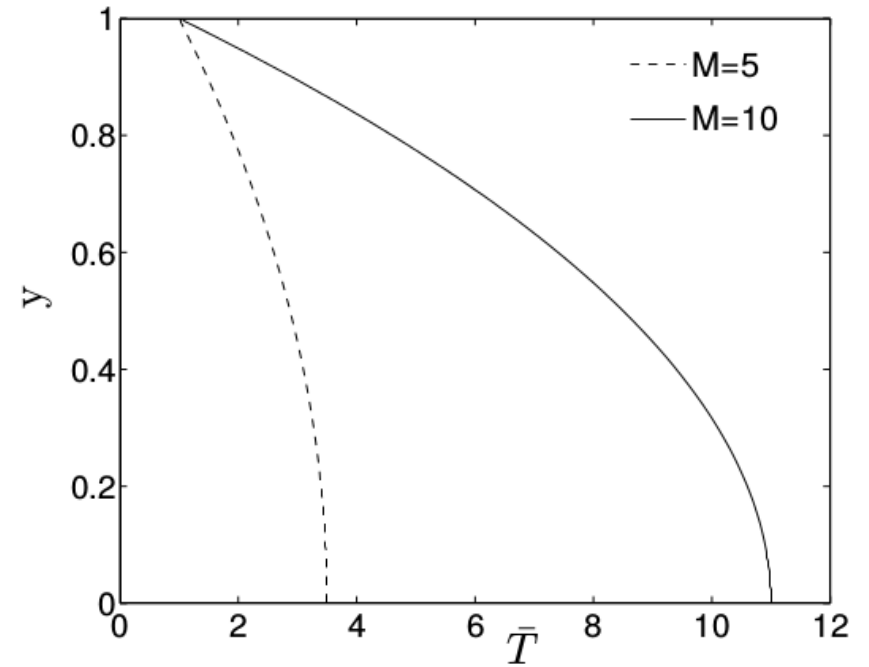
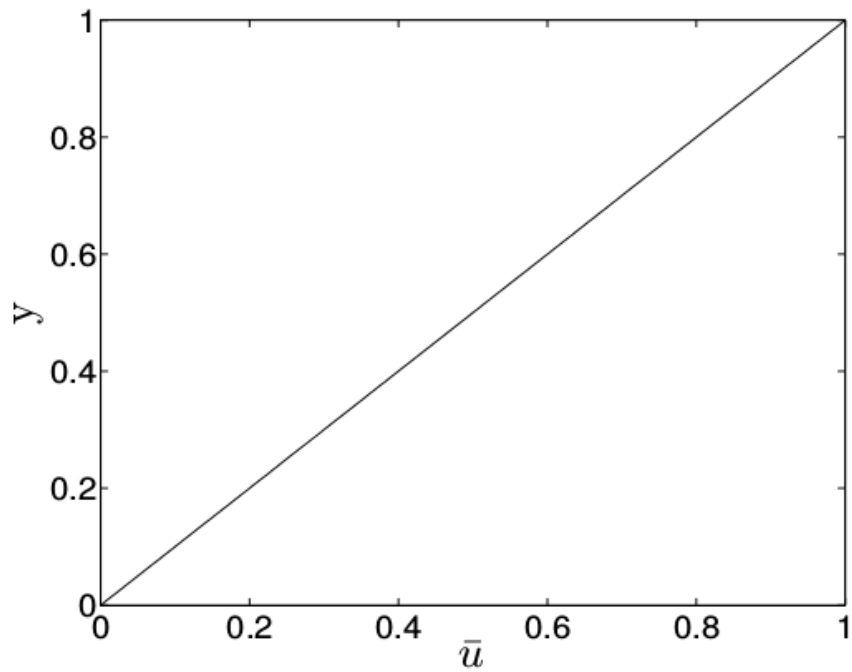


Fig: Variation of velocity & temperature in the wall - normal direction at $Pr = 0.5$

Stability Analysis

Linear stability equations

Steps for obtaining LS equations:

- Non-dimensionalize the equations

$$P^* = \rho^* R^* T^* \Rightarrow P = \rho T$$

- Separate the mean flow quantities into mean & perturbations

$$\bar{P} + \tilde{P} = (\bar{\rho} + \tilde{\rho}) (\bar{T} + \tilde{T}) = \bar{\rho}\bar{T} + \bar{\rho}\tilde{T} + \tilde{\rho}\bar{T} + \tilde{\rho}\tilde{T}$$

- Subtract the mean flow equation

$$\tilde{P} = \bar{\rho}\tilde{T} + \tilde{\rho}\bar{T} + \tilde{\rho}\tilde{T}$$

Linear stability equations

- Assume parallel flow ($\bar{A} = \bar{A}(y)$), neglect higher order & non-linear terms

$$\tilde{P} = \bar{\rho} \tilde{T} + \tilde{\rho} \bar{T}$$

- Assume a normal mode solution & substitute in the above equation

$$\tilde{A}(x, y, z, t) = \hat{A}(y) \exp^{i(\alpha x + \beta z - \omega t)}$$

$$\hat{P}(y) = \bar{\rho} \hat{T}(y) + \hat{\rho}(y) \bar{T}$$

Final stability equation

Disturbance frequency

Spanwise wavenumber

Streamwise wavenumber

Linear stability equations

- Convert the stability equations into an eigenvalue problem

$$\begin{bmatrix} B_{11} & B_{12} & \dots & B_{15} \\ B_{21} & B_{22} & \dots & B_{25} \\ \vdots & & \dots & \vdots \\ \vdots & & \dots & \vdots \\ B_{41} & B_{42} & \dots & B_{45} \\ B_{51} & B_{52} & \dots & B_{55} \end{bmatrix} \begin{Bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \\ \hat{\rho} \\ \hat{T} \end{Bmatrix} = \omega \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & \dots & \vdots \\ \vdots & & \dots & \vdots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{Bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \\ \hat{\rho} \\ \hat{T} \end{Bmatrix}$$

→ Eigenvalue of the problem

- Solve using Chebyshev spectral method using appropriate boundary conditions

Numerical method

Chebyshev collocation method :

- Collocation points are:

$$x_j = \cos\left(\frac{j\pi}{n}\right), \quad j = 0, 1, \dots, n$$

- Derivative matrix in the above points :

$$(D_N)_{00} = \frac{2N^2 + 1}{6}, \quad (D_N)_{jj} = -\frac{x_j}{2(1 - x_j^2)}, \quad j = 1, 2, \dots, N - 1$$

$$(D_N)_{NN} = -\frac{2N^2 + 1}{6}, \quad (D_N)_{ij} = -\frac{c_i(-1)^{i+j}}{c_j(x_i - x_j)}, \quad i \neq j, \quad i, j = 0, 1, \dots, N$$

$$c_i = \begin{cases} 2, & i = 0 \text{ or } N \\ 1, & \text{otherwise} \end{cases}$$

Stability Approach

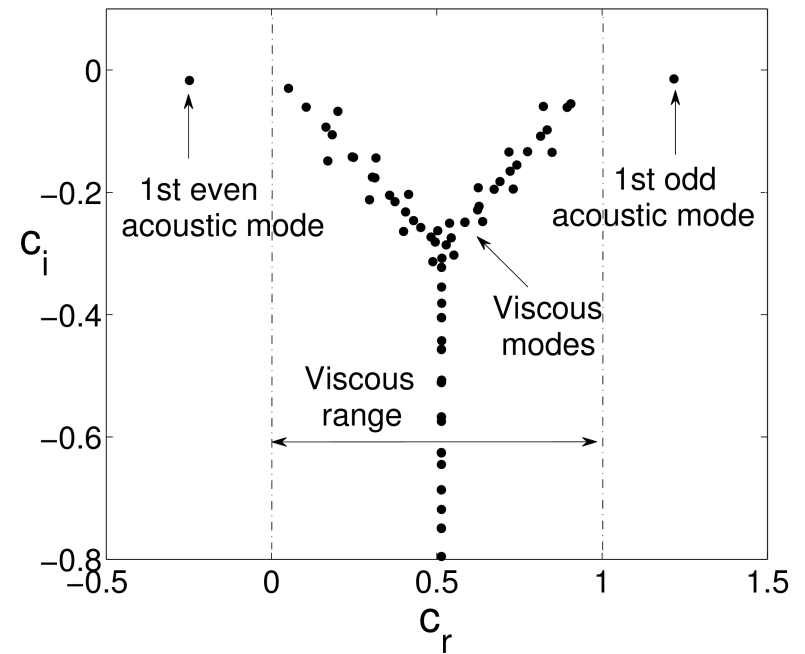
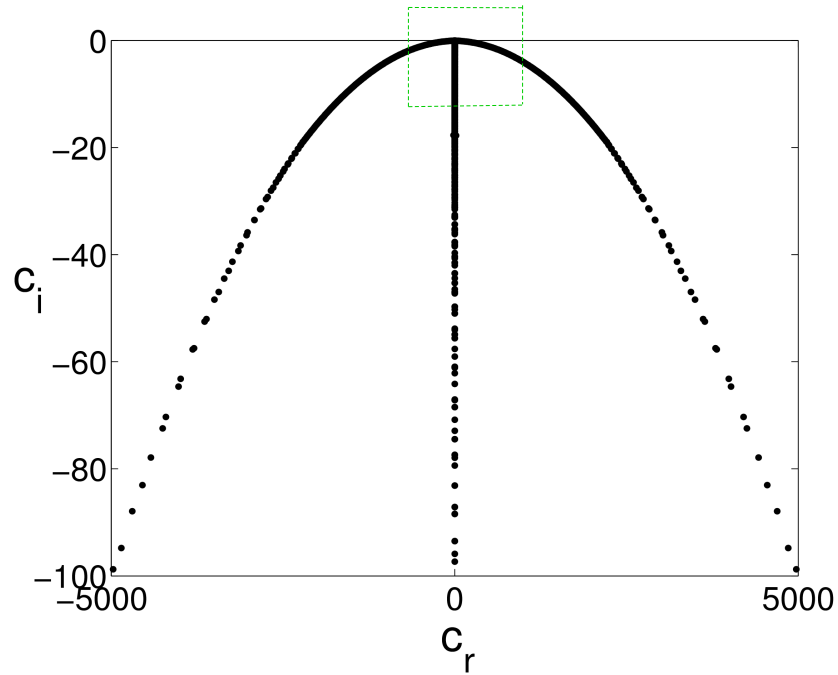
Linear temporal stability

$$e^{-i\omega t} = e^{-i(\omega_r + i\omega_i)t} = e^{-i\omega_r t} e^{\omega_i t}$$

Growth or decay rate

$\omega_i > 0$, disturbance grows

Types of Modes in the spectrum

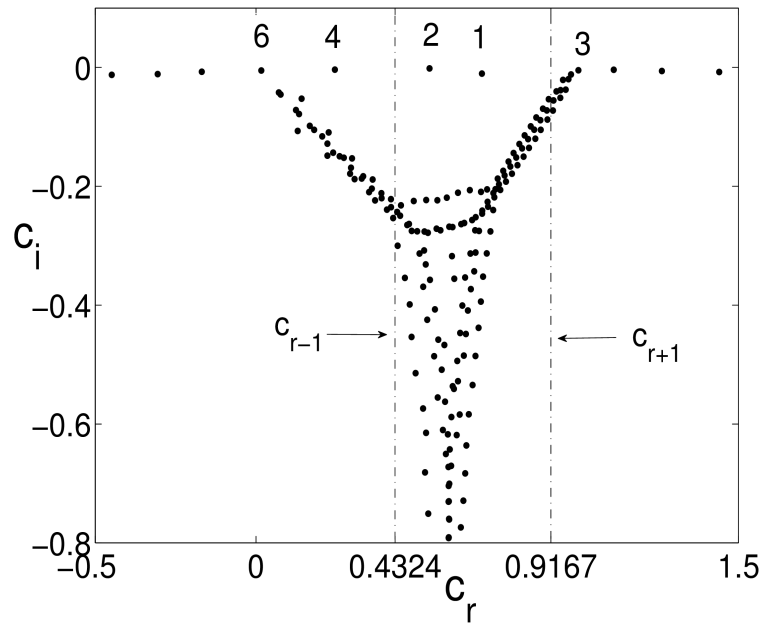


$M = 2$, $\alpha = 0.1$, $Re = 10^5$ and $Pr = 0.5$

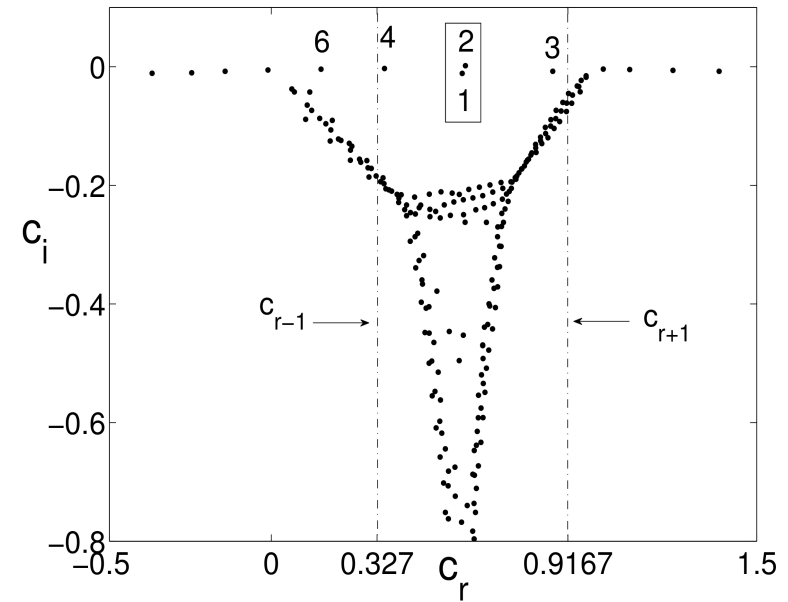
→ Acoustic and Viscous modes

Effect of Prandtl number

On the Eigenspectrum



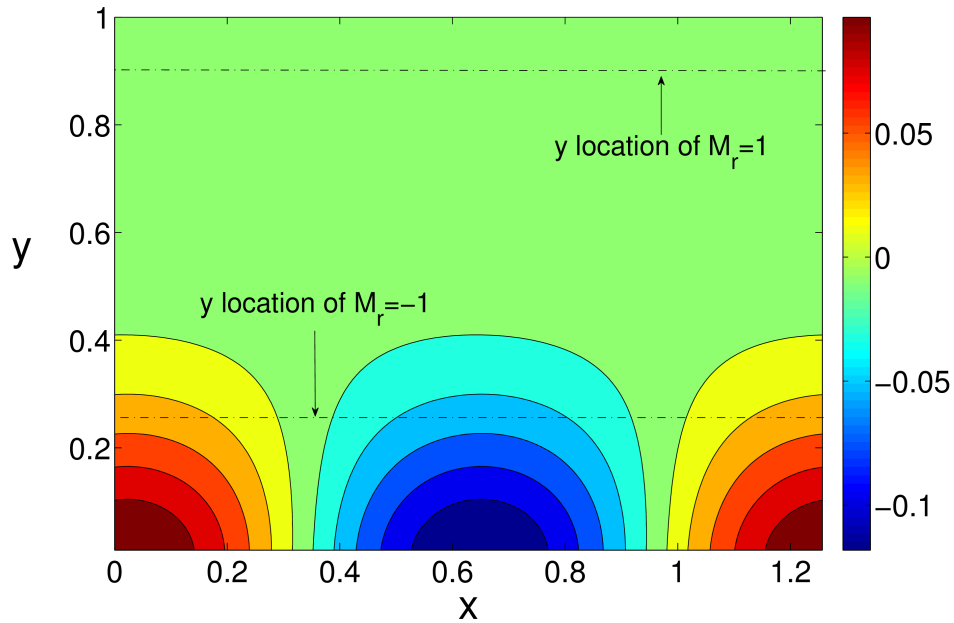
$Pr = 0.9$



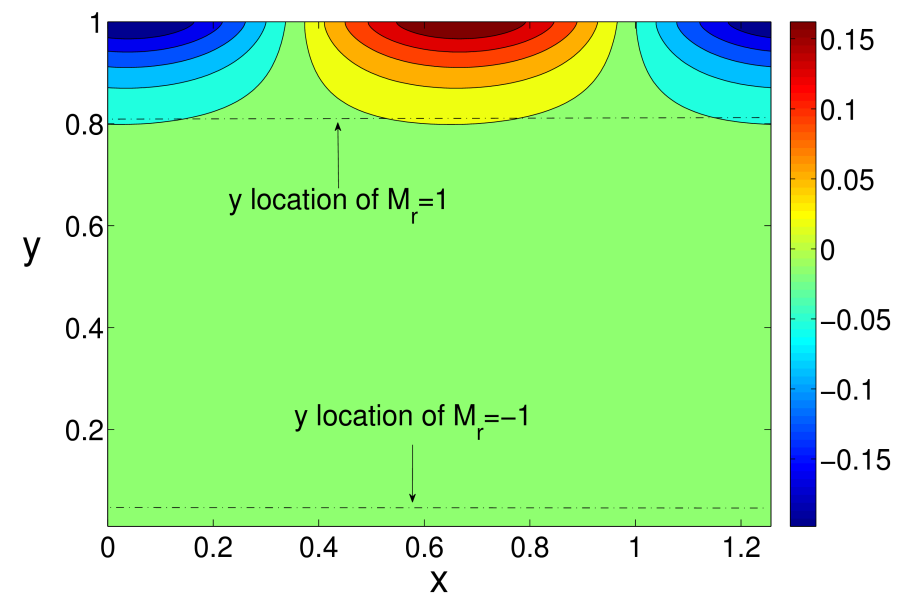
$Pr = 0.5$

- Forces the acoustic modes to enter into the viscous range
- Leads to **mode crossing** phenomenon

Regular pressure perturbation contours



Mode 1



Mode 2

Pr = 0.9

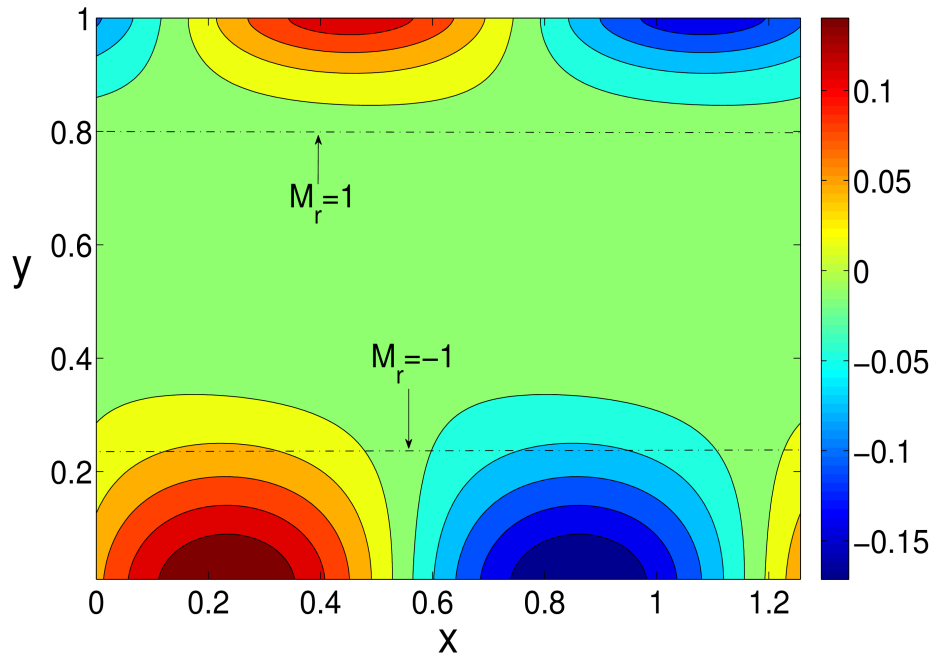
Relative Mach number :

$$M_r = \frac{\bar{u} - c_r}{\sqrt{\bar{T}}} M,$$

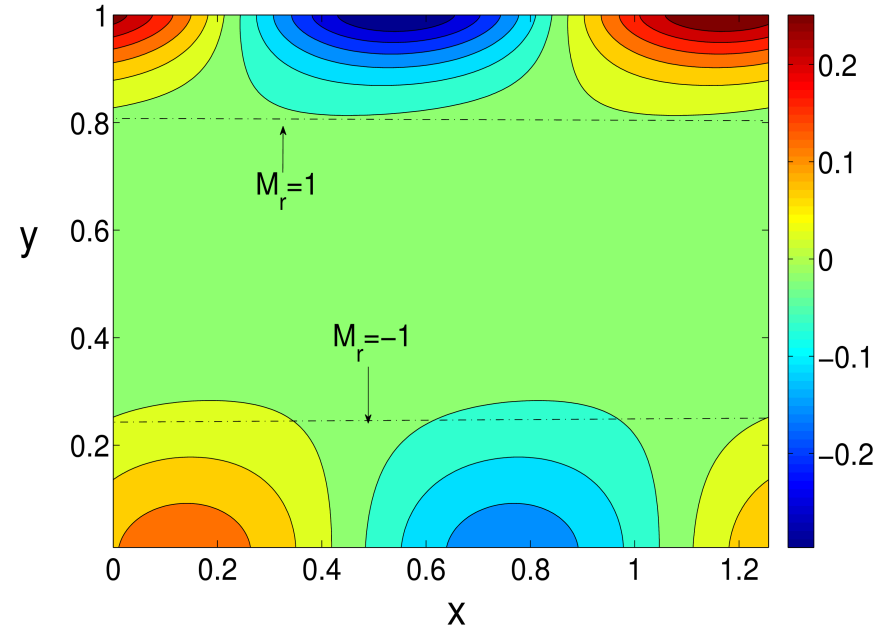
→ Odd mode: Bottom wall

→ Even mode: Top wall

During mode-crossing



Mode 1



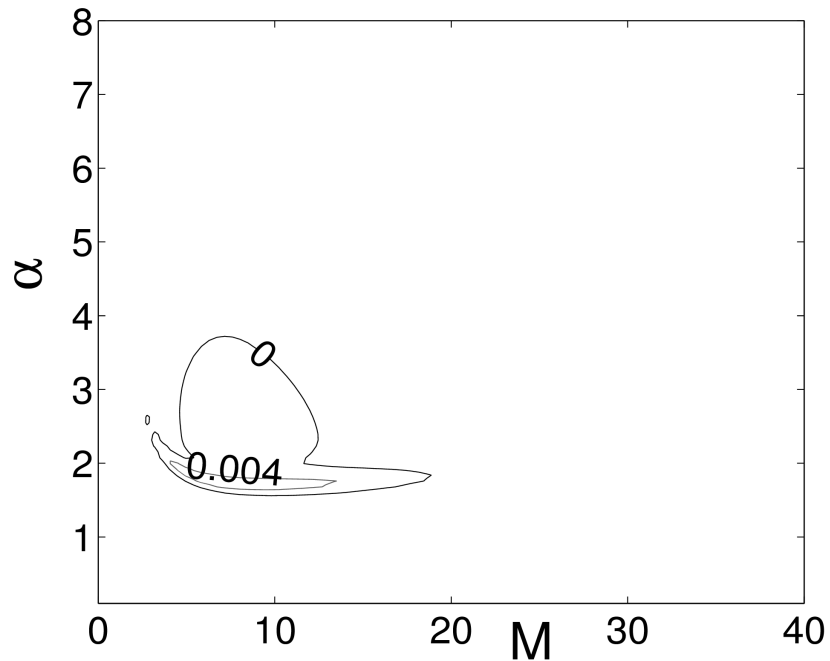
Mode 2

$Pr = 0.5$

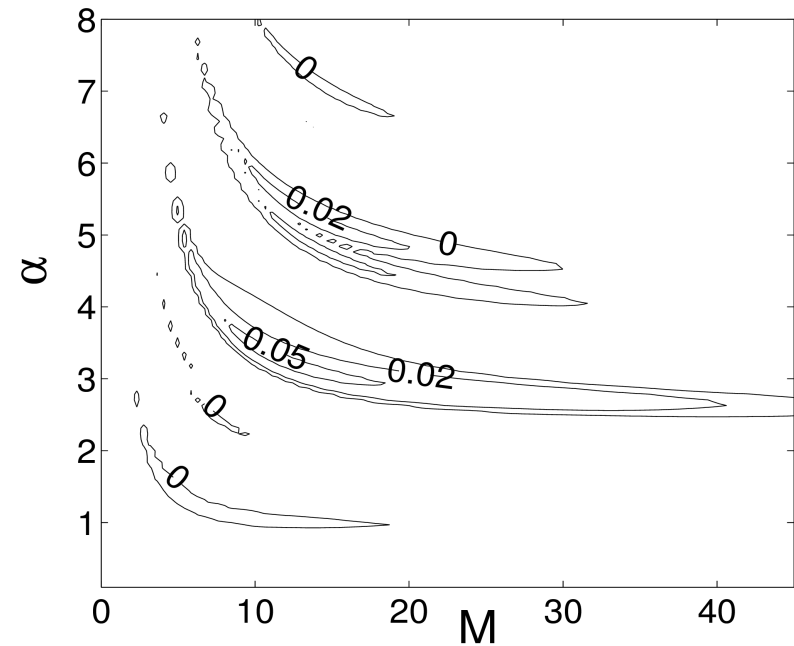
Due to mode-crossing :

- Odd mode: Bottom wall + Top wall due to mode 2
- Even mode: Top wall + Bottom wall due to mode 1

On the M-alpha diagram



$Pr = 0.7$

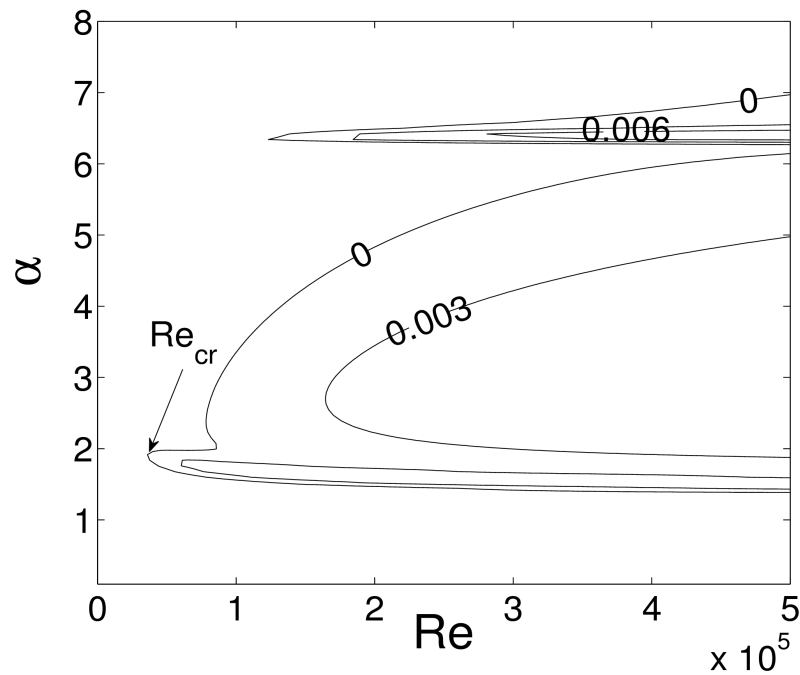


$Pr = 0.2$

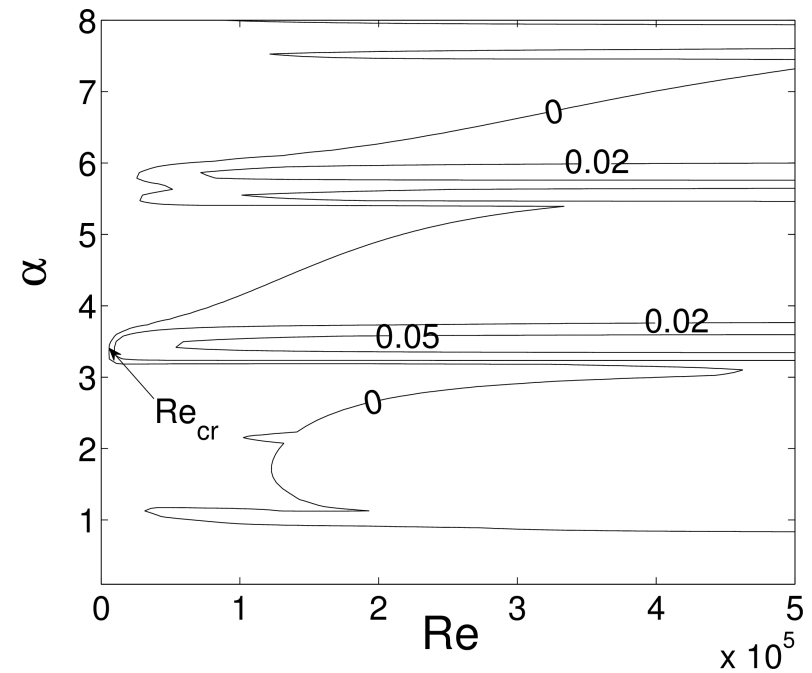
→ Additional Instability loops

→ Increase in size of the loops and growth rate values

On the Re-alpha diagram at $M = 10$



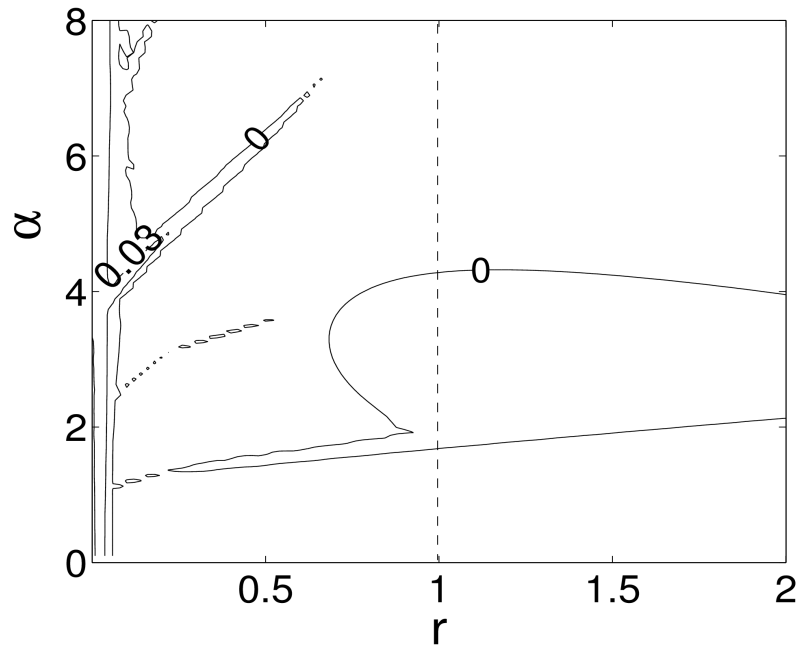
$Pr = 0.7$



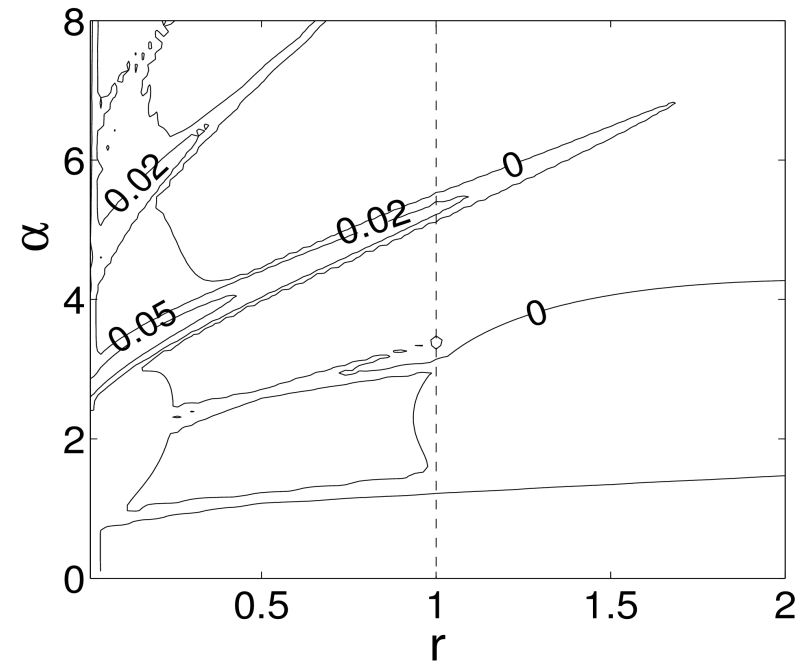
$Pr = 0.2$

	M=5		M=10	
Pr	Re_{cr}	α_{cr}	Re_{cr}	α_{cr}
0.7	19036	2.16	35680	1.90
0.5	15136	1.98	30042	1.66
0.2	10442	5.22	5232	3.34

On the r-alpha diagram at $M = 5$



$Pr = 0.7$



$Pr = 0.2$

$$r = \frac{T_w}{T_r}$$

$$T_r = 1 + \frac{(\gamma - 1) Pr M^2}{2}$$

Conclusion

As Prandtl number is reduced,

- More number of mode-crossings
- Additional instability loops
- Increase in maximum growth rate value
- Decrease in critical Reynolds number



Flow with a low Prandtl number is more unstable compared to a high Prandtl number flow

Future work

- Relate the acoustic modes of a wall-bounded flow to the boundary layer modes.
- Detailed analysis of mode-crossing phenomenon
- Energy analysis of selected acoustic modes
- Individual effect of viscosity and conductivity and their stratification on flow stability

THANK YOU !!!

Bibliography

- [1] A. Fedorov, "Transition and Stability of High-Speed Boundary Layers," Annual Review of Fluid Mechanics 43, 79 (2011).
- [2] X. Zhong and X. Wang, "Direct numerical simulation on the receptivity, instability, and transition of hypersonic boundary layers," Annual Review of Fluid Mechanics 44, 527 (2012).
- [3] E. Reshotko, "Hypersonic stability and transition," Hypersonic Flows for Reentry Problems 1, 18 (1991).
- [4] S. J. Leib, D. W. Wundrow and M. E. Goldstein, "Effect of free-stream turbulence and other vortical disturbances on a laminar boundary layer," Journal of Fluid Mechanics 380, 169 (1999).
- [5] M. R. Malik and D. I. A. Poll, "Effect of curvature on three-dimensional boundary-layer stability," AIAA 23 (9), 1362 (1985).
- [6] G. G. Mater, "The effect of angle of attack on boundary-layer transition on cones," AIAA 10 (8), 1127 (1972).
- [7] L. E. Ericsson, "Effect of nose bluntness and cone angle on slender-vehicle transition," AIAA 26 (10), 1168 (1988).

Bibliography

- [8] M. R. Malik, R. E. Spall, and C. L. Chang, "Effect of nose bluntness on boundary layer stability and transition," AIAA paper No. 1990-0112, 1990.
- [9] M. R. Malik and E. C. Anderson, "Real gas effects on hypersonic boundary-layer stability," *Physics of Fluids A: Fluid Dynamics* 3 (5), 803 (2013).
- [10] M. L. Hudson, N. Chokani and G. V. Candler, "Linear Stability of Hypersonic Flow in Thermo-chemical Nonequilibrium," *AIAA* 35 (6), 958 (1997).
- [11] T. W. Robarge and S. P. Schneider, "Laminar Boundary-Layer Instabilities on Hypersonic Cones: Computations for Benchmark Experiments," *AIAA Paper* No. 2005-5024, 2005.
- [12] H. B. Johnson and G. V. Candler, "Thermochemical interactions in the linear stability of hypersonic boundary layers," *AIAA Paper* No. 98-2438, 1998.
- [13] H. B. Johnson, T. G. Seipp and G. V. Candler, "Numerical study of hypersonic reacting boundary layer transition on cones," *Physics of Fluids* 10 (10), 2676 (1998).
- [14] K. J. Franko, R. W. MacCormack and S. K. Lele, "Effects of chemistry modeling on hypersonic boundary layer linear stability prediction," *AIAA Paper* No. 2010-4601, 2010.
- [15] I. J. Lyttle and H. L. Reed, "Sensitivity of second-mode linear stability to constitutive models within hypersonic flow," *AIAA Paper* No. 2005-889, 2005.

Bibliography

- [16] H. Alkandry, I. D. Boyd and A. Martin, "Comparison of Models for Mixture Transport Properties for Numerical Simulations of Ablative Heat-Shields," AIAA Paper 2013-0303, 2013.
- [17] B. V. Alexeev, A. Chikhaoui and I. T. Grushin, "Application of the generalized Chapman-Enskog method to the transport-coefficient calculation in a reacting gas mixture," Physical review E 49 (4), 2809 (1994).
- [18] G. K. Stuckert, PhD thesis, Dept. of Mechanical and Aerospace Engineering, Arizona State University, Tempe, 1991.
- [19] S. Hu and X. Zhong, "Linear stability of viscous supersonic plane Couette flow," Physics of Fluids 10 (3), 709 (1998).
- [20] M. Malik, J. Dey and M. Alam, "Linear stability, transient energy growth, and the role of viscosity stratification in compressible plane Couette flow," Physical review E 77 (3) (2008).
- [21] P. W. Duck, G. Erlebacher and M. Y. Hussaini, "On the Linear Stability of Compressible Plane Couette Flow," Journal of Fluid Mechanics 258, 131 (1994).
- [22] L. C. Scalabrin and I. D. Boyd, "Numerical simulations of the FIRE-II convective and radiative heating rates," AIAA Paper No. 2007- 4044, 2007.

Bibliography

- [23] M. I. Boulos, P. Fauchais and Emil Pfender, Thermal plasmas (Springer, New York, 1994), Vol. 1, pp. 265-321.
- [24] M. Capitelli, G. Colonna, C. Gorse and A. D'Angola, "Transport properties of high temperature air in local thermodynamic equilibrium," The European Physical Journal D-Atomic, Molecular, Optical and Plasma Physics 11 (2), 279 (2000).
- [25] M. R. Malik, "Numerical methods for hypersonic boundary layer stability," Journal of Computational Physics 86 (2), 376 (1990).
- [26] L. N. Trefethen, Spectral methods in MATLAB (Siam, Philadelphia, 2000), Vol. 10, pp. 51-55.
- [27] L. M Mack, "On the inviscid acoustic-mode instability of supersonic shear flows," Theoretical and Computational Fluid Dynamics 2 (2), 97 (1990).
- [28] W. Glatzel, "The linear stability of viscous compressible plane Couette flow," Journal of Fluid Mechanics 202, 515 (1989).
- [29] A. Federov and Anatoli Tumin, "High-speed boundary-layer instability: old terminology and a new framework," AIAA 49 (8), 1647 (2011).

Non-dimensionalization

$$(\bar{u}, \bar{v}, \bar{w}) = \frac{(\bar{u}^*, \bar{v}^*, \bar{w}^*)}{U_\infty}, \quad (x, y, z) = \frac{(x^*, y^*, z^*)}{L_\infty}, \quad t = \frac{U_\infty t^*}{L_\infty},$$

$$(\bar{\mu}, \bar{\lambda}) = \frac{(\bar{\mu}^*, \bar{\lambda}^*)}{\mu_\infty}, \quad \bar{T} = \frac{\bar{T}^*}{T_\infty}, \quad \bar{\rho} = \frac{\bar{\rho}^*}{\rho_\infty}, \quad \bar{p} = \frac{\bar{p}^*}{\rho_\infty R T_\infty}, \quad \bar{\kappa} = \frac{\bar{\kappa}^*}{\kappa_\infty},$$

$$Re = \frac{\rho_\infty U_\infty L_\infty}{\mu_\infty}, \quad Pr = \frac{\mu_\infty C_p}{\kappa_\infty}, \quad M = \frac{U_\infty}{\sqrt{\gamma R T_\infty}}.$$

NS equations in non-dimensional form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho \left(\frac{\partial u}{\partial t} + \mathbf{V} \cdot \nabla u \right) = -\frac{1}{\gamma M^2} \frac{\partial p}{\partial x} + \frac{1}{Re} \left\{ \frac{\partial}{\partial x} \left[(\lambda + 2\mu) \frac{\partial u}{\partial x} + \lambda \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right] + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[\mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right] \right\}$$

$$\rho \left(\frac{\partial v}{\partial t} + \mathbf{V} \cdot \nabla v \right) = -\frac{1}{\gamma M^2} \frac{\partial p}{\partial y} + \frac{1}{Re} \left\{ \frac{\partial}{\partial x} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[(\lambda + 2\mu) \frac{\partial v}{\partial x} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \right] + \frac{\partial}{\partial z} \left[\mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right] \right\}$$

NS equations in non-dimensional form

$$\rho \left(\frac{\partial w}{\partial t} + \mathbf{V} \cdot \nabla w \right) = -\frac{1}{\gamma M^2} \frac{\partial p}{\partial z} + \frac{1}{Re} \left\{ \frac{\partial}{\partial x} \left[\mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) \right] \right. \\ \left. + \frac{\partial}{\partial z} \left[(\lambda + 2\mu) \frac{\partial w}{\partial z} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right\}$$

$$\rho \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = (1 - \gamma) p \nabla \cdot \mathbf{V} + \frac{1}{Re Pr} [\nabla \cdot (\kappa \nabla T)] + \frac{(\gamma - 1) M^2}{Re} \Phi$$

$$\Phi = \mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] + 2\lambda \left[\frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \frac{\partial w}{\partial z} + \frac{\partial v}{\partial y} \frac{\partial w}{\partial z} \right] \\ + 2\mu \left[\frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial z} \frac{\partial w}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial w}{\partial y} \right] + (\lambda + 2\mu) \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right]$$

Non-dimensional linear stability equations

$$(u, v, w) = (\bar{u}(y), 0, 0) + (u'(x, y, z, t), v'(x, y, z, t), z'(x, y, z, t)),$$

$$\rho = \bar{\rho}(y) + \rho', \quad h = \bar{h}(y) + h', \quad T = \bar{T}(y) + T', \quad k = \bar{k}(y) + k', \quad \mu = \bar{\mu}(y) + \mu'$$

$$\frac{\partial \rho'}{\partial t} + \bar{\rho} \frac{\partial u'}{\partial x} + \bar{u} \frac{\partial \rho'}{\partial x} + \bar{\rho} \frac{\partial v'}{\partial y} + v' \frac{d\bar{\rho}}{dy} + \bar{\rho} \frac{\partial w'}{\partial z} = 0$$

$$\begin{aligned} \bar{\rho} \left(\frac{\partial u'}{\partial t} + \bar{u} \frac{\partial u'}{\partial x} + v' \frac{d\bar{u}}{dy} \right) = & -\frac{\partial p'}{\partial x} + \frac{1}{Re} \frac{\partial}{\partial x} \left[(\bar{\lambda} + 2\bar{\mu}) \frac{\partial u'}{\partial x} + \bar{\lambda} \left(\frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} \right) \right] \\ & + \frac{1}{Re} \left\{ \frac{\partial}{\partial y} \left[\mu' \frac{d\bar{u}}{dy} + \bar{\mu} \left(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right) \right] + \bar{\mu} \frac{\partial}{\partial z} \left[\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right) \right] \right\} \end{aligned}$$

Non-dimensional linear stability equations

$$\bar{\rho} \left(\frac{\partial v'}{\partial t} + \bar{u} \frac{\partial v'}{\partial x} \right) = -\frac{\partial p'}{\partial y} + \frac{1}{Re} \frac{\partial}{\partial y} \left[(\bar{\lambda} + 2\bar{\mu}) \frac{\partial v'}{\partial y} + \bar{\lambda} \left(\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} \right) \right] \\ + \frac{1}{Re} \left\{ \frac{\partial}{\partial x} \left[\bar{\mu} \left(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right) + \mu' \frac{d\bar{u}}{dy} \right] + \frac{\partial}{\partial z} \left[\bar{\mu} \left(\frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right) \right] \right\}$$

$$\bar{\rho} \left(\frac{\partial w'}{\partial t} + \bar{u} \frac{\partial w'}{\partial x} \right) = -\frac{\partial p'}{\partial z} + \frac{1}{Re} \left\{ \bar{\mu} \frac{\partial}{\partial x} \left[\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[\bar{\mu} \left(\frac{\partial w'}{\partial y} + \frac{\partial v'}{\partial z} \right) \right] \right\} \\ + \frac{1}{Re} \frac{\partial}{\partial z} \left[(\bar{\lambda} + 2\bar{\mu}) \frac{\partial w'}{\partial z} + \bar{\lambda} \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) \right]$$

Non-dimensional linear stability equations

$$\bar{\rho} \left(\frac{\partial h'}{\partial t} + \bar{u} \frac{\partial h'}{\partial x} + v' \frac{\partial \bar{h}}{\partial y} \right) = (\gamma - 1) M^2 \left\{ \frac{\partial p'}{\partial t} + \bar{u} \frac{\partial p'}{\partial x} \right\} + \frac{1}{Re Pr} \left\{ \bar{k} \left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial z^2} \right) + \frac{\partial}{\partial y} \left(\bar{k} \frac{\partial T'}{\partial y} + k' \frac{d\bar{T}}{dy} \right) \right\} \\ + \frac{(\gamma - 1) M^2}{Re} \left\{ 2\bar{\mu} \frac{d\bar{u}}{dy} \left(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right) + \mu' \left(\frac{d\bar{u}}{dy} \right)^2 \right\}$$

$$\frac{p'}{\bar{p}} = \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}.$$

$$\mu' = \frac{d\bar{\mu}}{d\bar{T}} T', \quad k' = \frac{d\bar{k}}{d\bar{T}} T'$$

Matrix entries

$$B_{11} = \alpha \bar{u} - \iota \frac{[\bar{\mu}(\alpha^2 + \beta^2) + \alpha^2(\bar{\mu} + \bar{\lambda}) - \bar{\mu}_y D - \bar{\mu} D^2]}{\bar{\rho} Re},$$

$$B_{12} = -\iota \bar{u}_y - \alpha \frac{[\bar{\mu}_y + (\bar{\lambda} + \bar{\mu}) D]}{\bar{\rho} Re},$$

$$B_{13} = -\iota \alpha \beta \frac{(\bar{\lambda} + \bar{\mu})}{\bar{\rho} Re},$$

$$B_{14} = \frac{\alpha \bar{T}^2}{\gamma M^2},$$

$$B_{15} = \frac{\alpha}{\gamma M^2} + \iota \frac{[\bar{u}_{yy} \bar{\mu}_T + \bar{u}_y \bar{T}_y \bar{\mu}_{TT} + \bar{u}_y \bar{\mu}_T D]}{\bar{\rho} Re},$$

$$B_{21} = -\alpha \frac{[\bar{\lambda}_y + (\bar{\lambda} + \bar{\mu}) D]}{\bar{\rho} Re},$$

$$B_{22} = \alpha \bar{u} + \iota \frac{[(\bar{\mu}_y + \bar{\lambda}_y) D - \bar{\mu}(\alpha^2 + \beta^2) + (\bar{\mu} + \bar{\lambda}) D^2 + \bar{\mu}_y D + \bar{\mu} D^2]}{\bar{\rho} Re},$$

$$B_{23} = -\beta \frac{[\bar{\mu}_y + (\bar{\lambda} + \bar{\mu}) D]}{\bar{\rho} Re},$$

Matrix entries

$$B_{24} = -\iota \frac{(\bar{T}_y + \bar{T}D)}{\bar{\rho}\gamma M^2},$$

$$B_{25} = -\iota \frac{(\bar{\rho}_y + \bar{\rho}D)}{\bar{\rho}\gamma M^2} - \frac{\alpha \bar{u}_y \bar{\mu}_T}{\bar{\rho}Re},$$

$$B_{31} = -\iota \alpha \beta \frac{(\bar{\lambda} + \bar{\mu})}{\bar{\rho}Re},$$

$$B_{32} = -\beta \frac{[\bar{\mu}_y + (\bar{\lambda} + \bar{\mu}) D]}{\bar{\rho}Re},$$

$$B_{33} = \alpha \bar{u} - \iota \frac{[\bar{\mu}(\alpha^2 + \beta^2) + \beta^2(\bar{\mu} + \bar{\lambda}) - \bar{\mu}_y D - \bar{\mu}D^2]}{\bar{\rho}Re},$$

$$B_{34} = \frac{\beta \bar{T}^2}{\gamma M^2},$$

$$B_{35} = \frac{\beta}{\gamma M^2},$$

$$B_{41} = \alpha \bar{\rho},$$

Matrix entries

$$B_{42} = -\iota (\bar{\rho}_y + \bar{\rho} D),$$

$$B_{43} = \beta \bar{\rho},$$

$$B_{44} = \alpha \bar{u},$$

$$B_{45} = 0,$$

$$B_{51} = \alpha (\gamma - 1) \bar{T} + \iota \frac{2(\gamma - 1) M^2 \bar{\mu} \bar{u}_y D}{\bar{\rho} Re},$$

$$B_{52} = -\iota \bar{T}_y - \iota (\gamma - 1) \bar{T} D - \frac{2\alpha (\gamma - 1) M^2 \bar{\mu} \bar{u}_y}{\bar{\rho} Re},$$

$$B_{53} = \beta (\gamma - 1) \bar{T},$$

$$B_{54} = 0,$$

$$B_{55} = \alpha \bar{u} + \iota \frac{[\bar{T}_{yy} \bar{k}_T + \bar{T}_y^2 \bar{k}_{TT} + 2\bar{T}_y \bar{k}_T D - (\alpha^2 + \beta^2) \bar{k} + \bar{k} D^2 + (\gamma - 1) M^2 Pr \bar{u}_y^2 \bar{\mu}_T]}{\bar{\rho} Re Pr}.$$

Relative sonic line

$$M_r = \frac{\bar{u} - c_r}{\sqrt{T}} M,$$

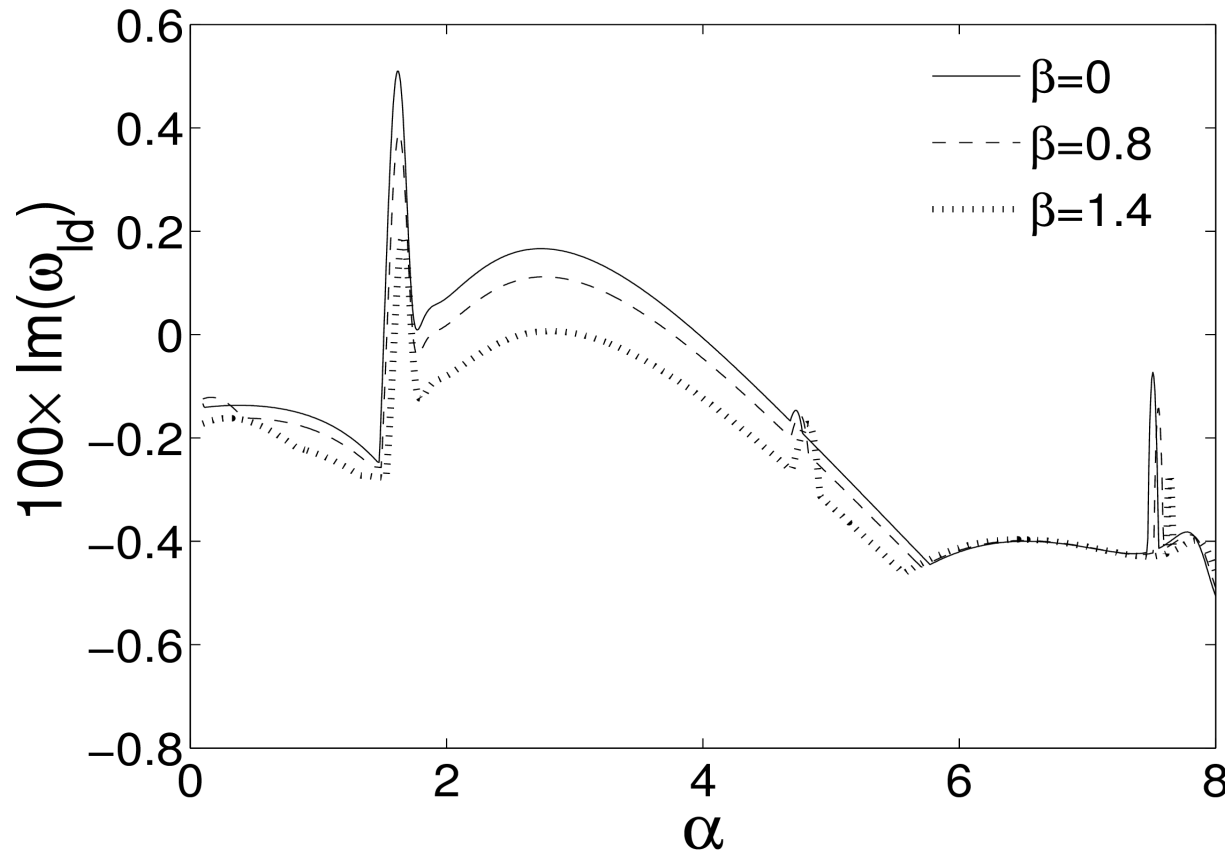
$$c_r \leq \bar{u} - \frac{\sqrt{T}}{M}, \text{ at the upper wall.}$$

$$c_{r+1} = 1 - \frac{1}{M}.$$

$$c_r \geq \bar{u} + \frac{\sqrt{T}}{M}, \text{ at the lower wall,}$$

$$c_{r-1} = \sqrt{\frac{1}{M^2} + \frac{(\gamma - 1)Pr}{2}}.$$

Comparison of 3D perturbations with 2D at $M = 5$



$Pr = 0.5$ and $Re = 2 * 10^5$

For some range of wavenumber:

Growth rate for 3D disturbance > 2D disturbance